

# Mathematical Culture: Genealogy

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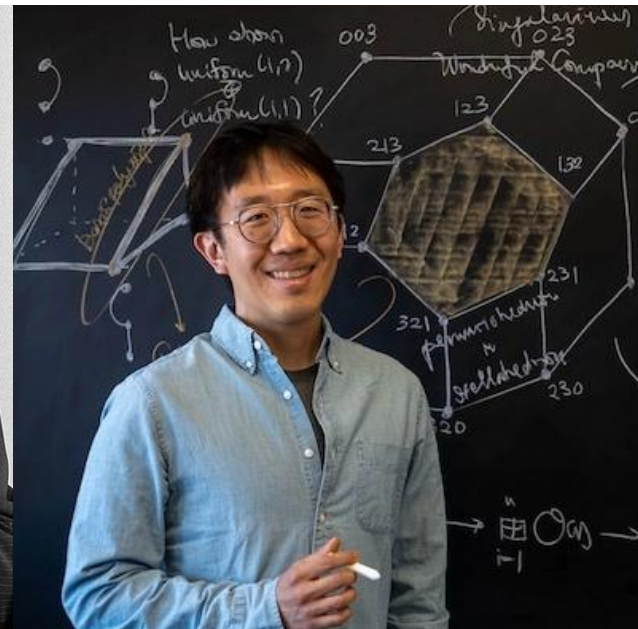
1. Mathematical Prizes
2. Mentors of Fields Medalists
3. “Three Crowns”
4. Heritage

# Mathematical Prizes

- Fields Medal (1936--): Outstanding discoveries in mathematics, CA\$15,000, every 4 years, age limit 40, awardees 64 so far
- Wolf Prize (1978--): Lifetime achievement, US\$100,000, every year, average age around 65, Youngest 42 (Andrew Wiles 1995), Oldest 82 (Carl L. Siegel 1978), awardees 69 so far
- Abel Prize (2003--): 6 million Norwegian kroner (NOK) =\$1 million “matching” the money amount of the Nobel Prize, every year, average age around 76, Youngest 63 (Andrew Wiles 2016), Oldest 90 (Louis Nirenberg 2015), awardees 28 till 2025
- Nobel Prizes do not include mathematics

# June E Huh: 2022 Fields Medalist

- Contribution: Introducing Hodge Theory into Combinatorics
- Mentor: 広中平佑 (Hironaka Heisuke) Fields Medal (1970) resolving singularities of algebraic varieties. Visited Korea to teach Algebraic Geometry course in 2007. He visited China and taught the same course at Huazhong University of Science and Technology in 1986. Elected to [Japan Academy](#), the [American Academy of Arts and Sciences](#) and academies in France, Russia, Korea and Spain.
- Unique relationship: Bole and the Thousand-Mile Horse
- Interdisciplinary



# Maryna S. Viazovska: 2022 Fields Medalist

- Ukrainian mathematician, the second female awardee (the first one 2014 [Maryam Mirzakhani](#) passed away) Salem Prize (2016) Clay Research Award (2017), [Ramanujan Prize](#) (2017), [New Horizons in Mathematics Prize](#) (2018), Fermat Prize (2019), ... ICM Speaker [\(2018\)](#), [Academia Europaea \(2021\)](#), [International Mathematical Olympiad](#) first place winner at age 16
- **Adviser: Don Bernard Zagier:** Putnam Fellow 16, Ph.D. 20, Habilitation 23, Full professor 24, Member of National Academy of Sciences (2017) and several other Academies
- Academic bother: Maxim L. Kontsevich, Fields Medal (1998), [Breakthrough Prize in Fundamental Physics](#) (2012) [Breakthrough Prize in Mathematics](#) (2015)



# 2 Fields Medal Students “Owners”

- Henri P. Cartan, Wolf Prize 1980, Students: Jean–Pierre Serre and Rene F. Thom received Fields Medal in 1954 and 1958, respectively.
- **Oscar Zariski**, Wolf Prize 1981, National Medal of Science 1965. Students: Hironaka Heisuke and David Mumford received Fields Medal in 1970 and 1974, respectively
- Elias M. Stein, Wolf Prize 1999, National Medal of Science 2001, Students: Charles Fefferman and Terence Tao received Fields Medal in 1978 and 2006, respectively
- Raoul Bott, Wolf Prize 2000, National Medal of Science 1987, Students: Stephen Smale and Denial Quillen received Fields Medal in 1966 and 1978, respectively
- **Don Bernard Zagier**, member of National Academy of Sciences, Students: Maxim L. Kontsevich and Maryna S. Viazovska received Fields Medals in 1998 and 2022, respectively
- Gérard Laumon, Clay Research Award 2004, Students: Laurent Lafforgue received Fields Medal in 2000 and Clay Research Award in 2002, **Ngô Bảo Châu 吴宝珠** received Clay Research Award in 2004 and Fields Medal in 2010.
- An interesting phenomenon: Most mentors of two Fields Medal students had Wolf Prize, and over half of them won National Medal of Science.

# 3 Generations Fields families

- Laurent–Moise Schwartz (1950)
- Alexander Grothendieck (1966)
- Pierre Deligne (1978)
  
- Pierre–Louis Lions (1994)
- Cédric Villani (2010)
- Alessio Figalli (2018)
  
- Paul J. Cohen (1966)
- Peter C. Sarnak (2014 Wolf Prize)
- Akshay Venkatesh (2018)

# “3 Crowns”<sup>1</sup> —Jean-Pierre Serre

- 1926.9.15, French mathematician
- Topology, algebraic geometry and number theory, Weil conjecture
- Fields Medal 1954 (27-28, the youngest)
- Wolf Prize 2000
- Abel Prize 2003 (The very first one)
- Adviser Henri P. Cartan, Wolf Prize 1980



# “3 Crowns”<sup>2</sup>—John Griggs Thompson

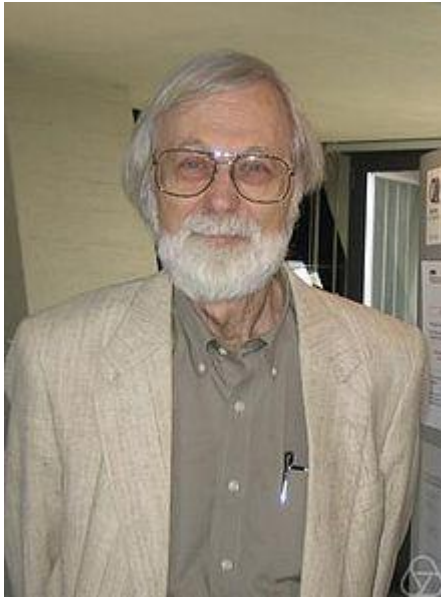
- 1932.10.13, University of Florida
- [classification of finite simple groups](#)
- [Fields Medal](#) 1970
- [Wolf Prize](#) 1992
- [Abel Prize](#) 2008
- Adviser Saunders Mac Lane, Vice president of [National Academy of Sciences](#) (1973-1981), President of [Mathematical Association of America](#) (1973–1981), President of American Mathematical Society (1973–1974), [National Medal of Science](#) (1989)







# “3 Crowns”3—John Willard Milnor



- 1931.2.20
- Distinguished professor [Stony Brook University](#)
- Differential topology, K-theory, and dynamical systems
- 《Topology from the differential viewpoint》 “Bible”
- [Fields Medal](#) 1962
- [Wolf Prize](#) 1989
- [Abel Prize](#) 2011
- [National Medal of Science](#) 1967
- [Leroy P Steele Prize](#) 1982, 2004, 2011
- Adviser: Ralph H. Fox, 1950 ICM Speaker
- “Grandpa” Solomon Lefschetz, made 《Annals of Mathematics》 the first class journal
- He visited WSU and delivered the Owens Lecture in 2009

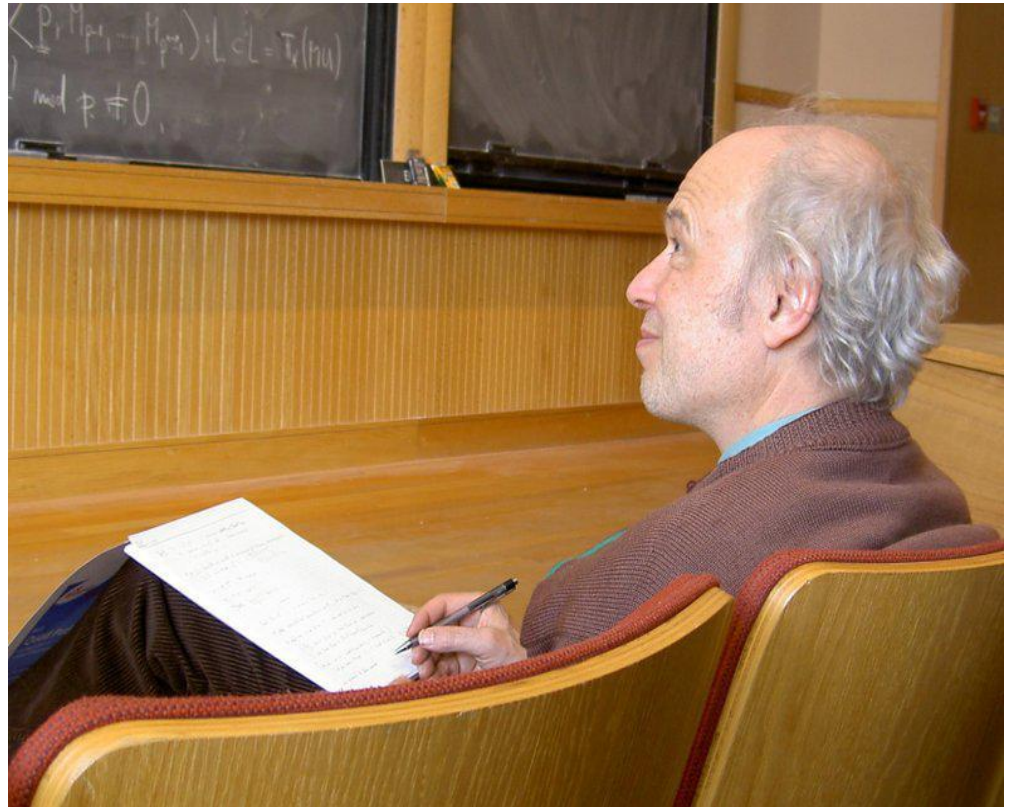
# John Willard Milnor (2009, Wayne State University)



# “3 Crowns”<sup>4</sup>

## Pierre René, Viscount Deligne

- 1944.10.3, Belgian, Weil conjectures
- Belgian King Albert II ennobled him as a viscount in 2006
- Collaborated with Jean-Pierre Serre and David Mumford
- Fields Medal 1978
- Wolf Prize 2008
- Abel Prize 2013
- Adviser: Grothendieck,  
“Pope” of algebraic geometry



# “3 Crowns”<sup>5</sup>—Grigory Margulis

- 1946, [Moscow, Russian American](#)
- Doctorate under Yakov Sinai (Wolf Prize & Abel Prize)
- **(Sinai’s adviser Andrey Kolmogorov)**
- [Member of National Academy of Sciences](#)
- Fields Medal 1978
- Wolf Prize 2005
- Abel Prize 2020
- Adviser Yakov G. Sinai received dozens of most influential mathematical award including Wolf Prize (1997) and Abel Prize (2014)
- Visited WSU and delivered the Owen’s lecture in 2018



# 3 Crowns” 5.5 -- Andrew Wiles

- 1953.4.11
- 1993.6.23 reported his results including the proof of Fermat’s Last Theory in Cambridge
- 1994.9, filled the gap with Richard Taylor, two articles published in May 1995 《Annals of Mathematics》
- 1995/6 Wolf Prize
- 1998 Fields Silver Medal
- 1999 the first Clay Research Award
- 2016 Abel Prize
- Adviser John Coates, fellow of the [Royal Society](#) (1985), President of the [London Mathematical Society](#) (1988-1990)
- Student Manjul Bhargava, 2014 Fields Medal



# Great Mathematicians produce extraordinary students?

- Some great mathematicians have not produced outstanding students.
- Reason 1: Died young, such as Abel (27), Galois (21), Ramanujan (32).
- Reason 2: Personality, such as Newton and Ramanujan.
- Reason 3: Extremely smart, others could not keep up with their thinking and students were discouraged. Poincaré and von Neumann.
- Reason 4: Too busy for their own research, as some “3 Crowns” owners
- But great mathematicians must have the soil that produces them.



# Great Mathematicians produce extraordinary students?

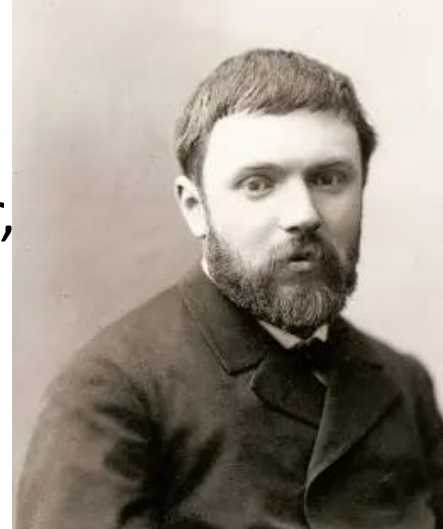
- Newton (1642-1726) seems to have no direct students.
- Roger Cotes (1682-1716) was not a student of Newton, he assisted in proofreading the second edition of Principia and invented the Newton-Cotes integral formula.
- William Whiston (1667-1752), theology, history, philosophy, mathematics. First time met Newton in 1694.
- These two were merely collaborators under his guidance, helping to organize and publish his works.

# Newton's Teachers?

- “Standing on the shoulders of giants.” who are these giants? There are two groups: ancient philosophers like Aristotle, Descartes, Galileo, Kepler (1571-1630), etc. contemporary teachers like John Wallis (1616-1703), Christiaan Huygens (1629-1695), Issac Barrow (1630-1677), Scotsman James Gregory (1638-1675)
- “The direction is more important than effort.” Newton spent much more time studying chemistry than physics and mathematics. Unfortunately, at that time, the field of chemistry had not yet accumulated enough for a revolutionary transformation. In his later years, he devoted to theology research.



# Henri Poincaré (1854-1912)



- French [mathematician](#), [theoretical physicist](#), engineer, and [philosopher of science](#). "The Last Universalist"
- "the [Gauss](#) of modern mathematics"
- Adviser: Charles Hermite (1822-1901)
- Academic brothers include Thomas Joannes Stieltjes (1856-1894) and Henri Padé (1863-1953)
- Hermite's academic "grandpa" Joseph Liouville (1808-1882)'s adviser Siméon Denis Poisson (1781-1840) was student of Lagrange
- Poincaré's students were not very famous, but his work influenced Albert Einstein and George David Birkhoff (1884-1944)
- Poincaré was extremely clever, and it was difficult for others to keep up with his thinking

# John von Neumann (1903-1957)

- Hungarian and American [mathematician](#)
- [physicist](#), [computer scientist](#), and [engineer](#)
- "Decathlon champion" among scientists
- Worked in the [Manhattan Project](#)
- IAP Fellow Among early 6 (including Einstein)
- Several PhD students were not well-known
- Some of his other students became famous.
- Teachers: David Hilbert, [Hermann Weyl](#) .....
- Academic brothers [George Pólya](#) (1887-1985) [Gábor Szegő](#) (1895-1985) [Paul Erdős](#) (1913-1996)
- Von Neumann was as brilliant as Poincaré, 7 languages, remarkable brain, .....



# Great Mathematician produce great students



- Joseph-Louis Lagrange (1736-1813) was a student of Euler, but not in the strict sense, called epistolary correspondent. Lagrange's tutor was Italian physicist Giovanni Battista Beccaria (1716-1781).
- Lagrange, the successor of Euler, an Italian immigrant to France, made outstanding contributions to analysis, number theory, classical mechanics and celestial mechanics as a mathematician, physicist and astronomer. One of the founders of variational calculus, analytical mechanics laid the foundation for 19th-century mathematical physics. Newtonian mechanics - Lagrangian mechanics.
- Students include: Siméon Denis Poisson (1781-1840) Joseph Fourier (1768-1830)
- Descendant count: 149958 form two students: Poisson and Fourier
- Riemann and Richard Dedekind (1831-1916) were students of Gauss

# Kars Weierstrass (1815-1897)

- German mathematician, a classic figure in textbooks.
- Initially studied law, was expelled from school for being naughty and mischievous.
- Entered “Normal School” to study mathematics and prepared to become a high school teacher. Fortunately, he met Christoph Gudermann (1798-1852), who opened a course on elliptic (Abelian) functions with only one student.
- High school teacher, 30 hours of class per week: In addition to math, there were German, geography, history, physical education, etc.
- On the Theory of Abelian Functions (1854)

# Kars Weierstrass

- Outstanding mentor, supervised 47 doctoral students, cultivating an entire generation of mathematicians for Germany and the world; famous mathematicians among his students include:
  - Georg Cantor (1845-1918)
  - Wilhelm Killing (1847-1923)
  - Ferdinand Frobenius (1849-1917)
  - Hermann Schwarz (1843-1921)
  - Sofya Kovalevskaya (1850-1891)
- Tracing back the mathematical “family tree,” he has 44578 descendants who are mathematicians.



# David Hilbert (1862-1943)



- Adviser: Ferdinand von Lindemann (1852-1939) who proved  $\pi$  is a transcendental number (1882)
- Academic brother Hermann Minkowski (1864-1909) who was Albert Einstein's teacher
- Academic "grandpa" Felix Klein (1849-1925) Klein bottle
- Students include:
  - Richard Courant (1888-1972) the founder of the Courant Institute
  - Alfréd Haar (1885-1933) [Haar measure](#), [Haar wavelet](#), [Haar transform](#)
  - Emanuel Lasker (1868-1941) 6 world chess championships, hold the title for 27 years (1894-1921)
  - Erhard Schmidt (1876-1959) [Gram-Schmidt orthogonal process](#)
  - Hermann Weyl (1885-1955) Weyl's law, [asymptotic distribution of eigenvalues](#)
- Descendant count 43215
- Gottingen was the world center of mathematics before WWII starting from Gauss
- Family tree of Gauss: 123425 "Gauss's descendant count is about 78000"
- **The Mathematics Genealogy Project Comes of Age at Twenty-one** Colm Mulcahy , Notices of the AMS, 64-5 (2017)

# Andrey Nikolayevich Kolmogorov (1903-1987)

- Godfather of Russian mathematics
- students include:
- Vladimir Arnold (1937-2010) Wolf Prize (2001) KAM Theory
- Eugene Dynkin (1924-2014) [Dynkin diagram](#), [Dynkin system](#), [Dynkin's lemma](#)
- Israel Gelfand (1913-2009) Wolf Prize (1978)
- Per Martin-Löf (1942-) Sweden Academy member
- Valery Vasilevich Kozlov (1950-) Russian Academy member
- Andrei Monin (1927-2001) Russian Academy member
- Yakov Sinai (1935--) Wolf Prize (1997) Abel Prize (2014), student [Grigory Margulis](#) is one of the 5.5



# Jacques-Louis Lions (1928-2001)



- Adviser: Laurent Schwartz (1915-2002)  
Fields Medal (1950)
- Son: Pierre–Louis Lions (1956--)  
Fields Medal (1994)
- Academic brother [Alexander Grothendieck](#) (1928-2014) Fields medal (1966)
- “Godfather” of French applied mathematics, Students include (all members of different [Academy of Sciences](#)):
- Alain Bensoussan (1940--) Top 2% most highly cited scientists
- Jean-Michel Bismut (1948--) Roland Glowinski (1937--2022)
- Roger Meyer Temam (1940--) Ph.D. students 130, No.4
- [Erol Gelenbe](#) (1945--) Ph.D. Students 95, Rank No. 11



# Notices

- [C.-C. Jay Kuo](#) 178 (No.1)
- Kolmogorov 82
- Hilbert 78
- Tsing-Tung Yau 丘成桐 75 (Chinese No.1)
- [Jean-Claude Nédélec](#) 72
- Gang Tian 田刚 67
- [Stanley Joel Osher](#) 64
- Chi-Wang Shu 舒其望 57
- **The Mathematics Genealogy Project Comes of Age at Twenty-one**  
*Colm Mulcahy*, Notices of the AMS, 64-5 (2017)
- <https://www.mathgenealogy.org/most-students.php?count=175>

# Path of Inheritance

1. Immortal work
2. Masterpiece
3. The oral transmission of art and thought between master and disciple, as well as among peers.
4. “a few tricks”
5. .....

# “Revelation”

1. Direction is more important than effort
2. “Heroes emerge in clusters”
3. Mentors are important, as well as fellow students --- “peer influence”
4. Jump out of the comfortable zone, go to high level places
5. Learn the characteristics and essence; master several unique skills or “tricks”
6. ....

# References

- David M Bressoud 《The Queen of the Sciences: A History of Mathematics》 2008
- George Dyson 《Turing's Cathedral: The Origins of the Digital Universe》 2012
- Constance Reid 《Hilbert》 1996
- Wikipedia, Baidu
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- “八卦菲尔茨奖” 《数学文化》 2022 年13卷4期 77页

# Gottfried W. Leibniz (1646-1716)

- A polymath, studied with different teachers in various fields, mainly influenced by Christiaan Huygens in mathematics
- Most famous student in mathematics, [Jacob Bernoulli](#) (1655-1705, epistolary correspondent)
- The perfection of the mathematical theory of calculus was built on the framework and symbols created by Leibniz, with the expression being very important. It was later developed by several generations of mathematicians including Bernoulli and Euler.
- Descendant count: 174485, mainly due to Bernoulli and Euler
- Britain did not keep up with time due to the “invention rights” dispute between Newton and Leibniz.

# Students of Riemann (1826-1866)

- Eduard Selling (1834-1920), Obtained his doctorate in 1859 under Riemann, and later devoted himself to the study of mechanical calculators, which was not successful in mathematics. In 1891, he failed to apply for the position of full professor, and retired as an emeritus professor in 1906.
- Gastav Roch (1839-1866), Worked at Göttingen for three semesters (1861-1862), attended Riemann's lectures, but was not strictly speaking his doctoral supervisor. The Riemann-Roch theorem that made him famous is a well-known theorem in algebraic topology.